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1. Consider the following bivariate distribution 𝑝(𝑥, 𝑦) of two discrete random  
variables 𝑋 and 𝑌.

A grid of numbers and symbols

Description automatically generated  
Compute:  
a.The marginal distributions 𝑝(𝑥) and 𝑝(𝑦).

b. The conditional distributions 𝑝(𝑥|𝑌 = 𝑦1) and 𝑝(𝑦|𝑋 = 𝑥3)

2. Consider two variables 𝑥 and 𝑦 with joint distribution 𝑝(𝑥, 𝑦). Prove the  
following two results

𝐸[𝑥] = 𝐸𝑦[𝐸𝑥[𝑥|𝑦]]

Since E[X|Y] = , which is the expectation of the distribution corresponding to PMF p(x|y)

=

=

=

=

=

= E[x]

3. Prove the relationship: 𝑉𝑥 = 𝐸𝑋[𝑥2] - (𝐸𝑋[𝑥])2, which relates the standard  
definition of the variance to the raw-score expression for the variance.

var[*X*] = E

= E

= E

= E

4. In a study, physicians were asked what the odds of breast cancer would be in  
a woman who was initially thought to have a 1% risk of cancer but who ended  
up with a positive mammogram result (a mammogram accurately classifies  
about 80% of cancerous tumors and 90% of benign tumors). 95 out of a  
hundred physicians estimated the probability of cancer to be about 75%. Do  
you agree?

Denote the events as follow:

A: positive result

B: cancerous tumor

C: benign tumor

* P(A|B) = 0.8
* P(B) = 0.01 🡪

To verify the estimation, P(B|A) needs computing:

P(B|A) =

* Disagree with the statement above

5. You find yourself on a game show, and the host presents you with four doors.  
Behind three doors are an assortment of gummy bears, and the remaining door  
has a pure gold car behind it!  
You pick a door, and then the host reveals a door behind which he knows is  
only a couple of red gummy bears. Then you are given the choice to stick with  
your first choice or switch to one of the other two unopened doors.  
If the probability that you will win the car if you switch is , where 𝑎 and 𝑏  
are coprime positive integers, what is 𝑎 + 𝑏?

Let A denotes the event that the first door (door 1) chosen is the one having the car

Let B denotes the event that the host opens one of the other doors (door 2)

P(A|B) =

* As the host opens door 2, the probability of having the car behind door 1 is ¼ , thus the chance of the car being behind the other 2 doors is 1- ¼ = ¾
* a + b = 7

Explain :

There is a ¼ chance that the car is behind door 1, each of there 3 remaining door has a 1/3 chance of being choosen by the host

* If the car is behind door 1, the host will open door 2 (1/3)
* If the car is behind door 2, the host will open either door 3 or door 4 (1/2)